# Multi–Dimensional Tensor–Based Adaptive Filter (TBAF) for Low Dose X–Ray CT


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## Introduction

Edge-preserving adaptive filtering within CT image reconstruction is a powerful method to reduce image noise and hence to reduce patient dose. However, highly sophisticated adaptive filters typically comprise many parameters which must be adjusted carefully in order to obtain optimal filter performance and to avoid artifacts caused by the filter. In this work we applied an anisotropic tensor-based adaptive image filter (TBAF) to CT image reconstruction, both as an image-based post-processing step, as well as a regularization step within an iterative reconstruction. The TBAF is a generalization of the filter of reference [1]. Provided that the image noise (i.e. the variance) of the original image is known for each voxel, we adjust all filter parameters automatically except for one single parameter *s* which can be interpreted as global filter strength. Hence, the TBAF can be applied to any individual CT dataset without user interaction. This is a crucial feature for a possible application in clinical routine. The TBAF is compared to a well-established adaptive bilateral filter using the same noise adjustment.

## Method

The mathematical background of the *N*-dim TBAF is shown in Fig. 1. Note that the filters $g_{kl}(\mathbf{r})$ are shift-invariant since all local information is contained solely in the weights $w_{kl}$. Hence, the convolutions $f(\mathbf{r}) * g_{kl}(\mathbf{r})$ $\forall k \leq l$ can be performed in a pre-processing step and the actual filtering is just a linear combination of the $N(N+1)/2$ convolved images with local weights.

## Results and Discussion

We applied the bilateral reference filter and the proposed TBAF both as an image-based post-processing step, as well as a regularization step within an iterative OSSART reconstruction. As “original” noisy image we took the image from an OSSART reconstruction with 3 iterations without regularization. The patient rawdata were taken from an ultra low-dose scan of a Siemens Definition Flash CT scanner. Fig. 2 shows a transversal and a coronal slice of the full original dataset.

Fig. 3 shows the results for the image-based filters. The filtered images show significant noise reduction without loss of anatomical details. Thereby, the TBAF filtered images look very similar to the bilateral filtered images. This demonstrates that the complex TBAF is well adjusted and does not show artifacts due to awkward parameter settings.

**Local orientation tensor of an image $f(\boldsymbol{r})$ in $N \geq 2$ dimensions:**

$$O_{mn}(\boldsymbol{r}) = \int_{\mathbb{R}^N} [\partial_m f(\boldsymbol{s})]\, [\partial_n f(\boldsymbol{s})]\, k(\boldsymbol{r}-\boldsymbol{s})\, d^N s\,, \quad 1 \leq m, n \leq N,$$

with Eigenvalues $\lambda_1 \geq \ldots \geq \lambda_N$,

and orthonormal Eigenvectors $\boldsymbol{o}_1, \ldots, \boldsymbol{o}_N$, $\quad \boldsymbol{o}_n \cdot \boldsymbol{o}_m = \delta_{mn}$.

$k(\boldsymbol{r})$ is an appropriate smoothing kernel.

**TBAF in Fourier domain:**

$$G(\boldsymbol{u}) = 1 - G_{\text{HP}}(u) \sum_{n=1}^{N} h(\lambda_n) \frac{(\boldsymbol{u} \cdot \boldsymbol{o}_n)^2}{u^2}. \qquad h(\lambda) = \begin{cases} 1 & \text{for} \quad \lambda < t_0, \\ \dfrac{t_1 - \lambda}{t_1 - t_0} & \text{for } t_0 \leq \lambda < t_1, \\ 0 & \text{for } t_1 \leq \lambda. \end{cases}$$

$G_{\text{HP}}(u)$ is an appropriate 1D high-pass filter in Fourier domain.

$t_0(\boldsymbol{r}) < t_1(\boldsymbol{r})$ are locally adapted thresholds (see below).

**TBAF in spatial domain:**

$$g(\boldsymbol{r}) = \mathsf{F}^{-1}[G(\boldsymbol{u})](\boldsymbol{r}) \;=\; \delta(\boldsymbol{r}) - \sum_{k \leq l}^{N} w_{kl}\, g_{kl}(\boldsymbol{r})$$

$$\text{with} \;\; g_{kl}(\boldsymbol{r}) = \mathsf{F}^{-1}\left[\frac{u_k\, u_l}{u^2} G_{\text{HP}}(u)\right](\boldsymbol{r})\,, \quad w_{kl} = \sum_{n=1}^{N} h(\lambda_n)\,(2 - \delta_{kl})\, o_{nk}\, o_{nl}\,.$$

**TBAF filtered image $\tilde{f}(\boldsymbol{r})$:**

$$\tilde{f}(\boldsymbol{r}) \;=\; f(\boldsymbol{r}) * g(\boldsymbol{r}) \;=\; f(\boldsymbol{r}) - \sum_{k \leq l}^{N} w_{kl}\, [f(\boldsymbol{r}) * g_{kl}(\boldsymbol{r})]\,.$$

**Adaptive bilateral filter as reference:**

$$\tilde{f}(\boldsymbol{r}) = \frac{\int_{\mathbb{R}^N} f(\boldsymbol{s})\, D(\boldsymbol{r},\boldsymbol{s})\, R(\boldsymbol{r},\boldsymbol{s})\, d^N s}{\int_{\mathbb{R}^N} D(\boldsymbol{r},\boldsymbol{s})\, R(\boldsymbol{r},\boldsymbol{s})\, d^N s}$$

$$D(\boldsymbol{r},\boldsymbol{s}) = \exp\left(-\frac{(\boldsymbol{r}-\boldsymbol{s})^2}{2\sigma_d^2}\right), \;\; R(\boldsymbol{r},\boldsymbol{s}) = \exp\left(-\frac{(f(\boldsymbol{r}) - f(\boldsymbol{s}))^2}{2\sigma_r^2(\boldsymbol{r})}\right).$$

**Adaptive filter parameter settings:**

$\sigma_r^2(\boldsymbol{r}) = s^2\, \mathrm{Var} f(\boldsymbol{r}), \quad t_i(\boldsymbol{r}) = s_i\, s^2\, \mathrm{Var} f(\boldsymbol{r}),\; i = 0, 1.$

$s$ is a global filter strength parameter, $\mathrm{Var} f(\boldsymbol{r})$ is the estimated local image noise.

$s_0 < s_1$ were determined empirically such that the TBAF becomes comparable to the bilateral reference filter.

**Fig. 1:** Mathematical background of the multi-dimensional tensor-based adaptive filter (TBAF).

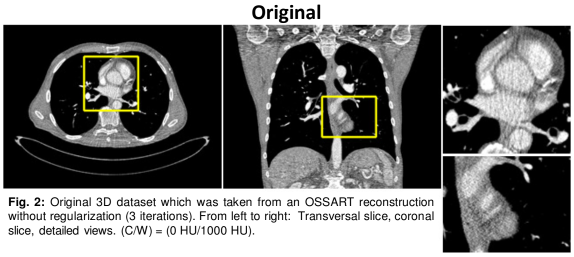


**Fig. 2:** Original 3D dataset which was taken from an OSSART reconstruction without regularization (3 iterations). From left to right: Transversal slice, coronal slice, detailed views. (C/W) = (0 HU/1000 HU).

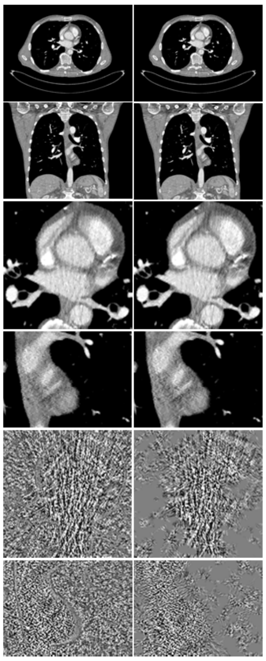


**Fig. 3:** Filters applied as an image-based post-processing step. Left: 3D bilateral filtered image, Right: 3D TBAF filtered image. From top to bottom, alternating transversal and coronal view: Full view, detailed view, detailed view of filtered minus original image.
Filtered images: (C/W) = (0 HU/1000 HU).
Difference images: (C/W) = (0 HU/100 HU).

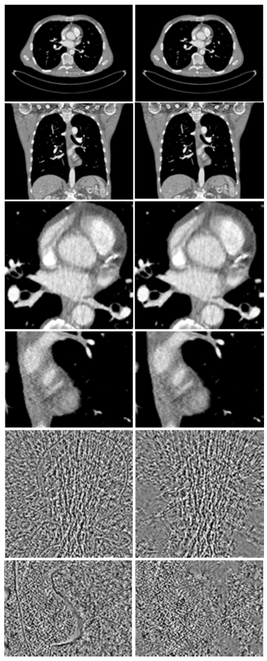


**Fig. 4:** Filters applied as a regularization step between the OSSART iterations (3 iterations, 2 regularization steps). Left: 3D bilateral filtered image, Right: 3D TBAF filtered image. From top to bottom, alternating transversal and coronal view: Full view, detailed view, detailed view of filtered minus original image.
Filtered images: (C/W) = (0 HU/1000 HU).
Difference images: (C/W) = (0 HU/100 HU).

On the other hand, there are visual differences between both filters which are hard to outline in the printout but can be clearly observed on an examination monitor. These differences are revealed by the difference images which compare the filtered images to the original image. In particular, edges and structures can be seen more clearly in the TBAF filtered images. Besides, it is notable that there are some image regions where the TBAF does not dare to filter at all.

Fig. 4 shows the results of an iterative reconstruction with the TBAF and the bilateral filter, resp. as a regularization step between the OSSART iterations. In each case, we performed 3 OSSART iterations and 2 regularization steps. Since the last step of the reconstruction was an OSSART iteration which restores the rawdata coverage of the filtered image, the differences between the TBAF and the bilateral filter are even less pronounced than in fig. 3. However, the difference images show that the TBAF only reduces image noise while the bilateral filter also affects some anatomical details, in particular at local edges.

## Conclusion

We applied an orientation tensor-based adaptive filter (TBAF) to low-dose CT image reconstruction and compared it to a well-established bilateral filter. Provided that the image noise (i.e. the variance) of the original image is known for each voxel, the adjustment of the filter parameters is done automatically and hence does not require any user interaction. The TBAF filtered images are very similar to the bilateral filtered images which demonstrates that the parameters of the TBAF are well adjusted. On the other hand, the TBAF better preserves the anatomical details which might be crucial for medical applications.



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