Basics of X-Ray-Based Tomographic Imaging for IGRT 2: CT Image Reconstruction

Marc Kachelrieß

German Cancer Research Center (DKFZ) Heidelberg, Germany www.dkfz.de/ct



Fan-Beam Geometry (transaxial / in-plane / x-y-plane)

x-ray tube



field of measurement (FOM) and object

detector (typ. 1000 channels)









Data Completeness





Each object point must be viewed by an angular interval of 180° or more. Otherwise image reconstruction is not possible.



V

Data Completeriess



Any straight line through a voxel must be intersected by the source trajectory at least once.



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Emission vs. Transmission

Emission tomography

- Infinitely many sources
- No source trajectory
- Detector trajectory may be an issue
- 3D reconstruction relatively simple

Transmission tomography

- A single source
- Source trajectory is the major issue
- Detector trajectory is an important issue
- 3D reconstruction extremely difficult



Analytical Image Reconstruction











Filtered Backprojection (FBP)

Measurement: $p(\vartheta, \xi) = \int dx dy f(x, y) \delta(x \cos \vartheta + y \sin \vartheta - \xi)$ Fourier transform: $\int d\xi p(\vartheta, \xi) e^{-2\pi i \xi u} = \int dx dy f(x, y) e^{-2\pi i u (x \cos \vartheta + y \sin \vartheta)}$

This is the central slice theorem: $P(\vartheta, u) = F(u\cos\vartheta, u\sin\vartheta)$ Inversion: $f(x,y) = \int_{0}^{\pi} d\vartheta \int_{0}^{\infty} du |u| P(\vartheta, u) e^{2\pi i u (x\cos\vartheta + y\sin\vartheta)}$ Important: K(0) = 0 and $K'(0\pm) = \pm 1$ $= \int_{0}^{\pi} d\vartheta p(\vartheta, \xi) * k(\xi) \Big|_{\xi = x\cos\vartheta + y\sin\vartheta}$



Filtered Backprojection (FBP)

Filter projection data with the reconstruction kernel.
 Backproject the filtered data into the image:



Smooth

Standard

Reconstruction kernels balance between spatial resolution and image noise.













CT Angiography: Axillo-femoral bypass

M = 4

120 cm in 40 s

0.5 s per rotation 4×2.5 mm collimation pitch 1.5









H. Bruder, M. Kachelrieß, S. Schaller. SPIE Med. Imag. Conf. Proc., 3979, 2000

CT-Angiography Sensation 64 spiral scan with 2·32×0.6 mm and 0.375 s







Flat Detector CT = Feldkamp-Type Reconstruction

Approximate

- Similar to 2D reconstruction:
 - row-wise filtering of the rawdata
 - followed by backprojection
- True 3D volumetric backprojection along the original ray direction
- Compared to ASSR:
 - larger cone-angles possible
 - lower reconstruction speed
 - requires 3D backprojection hardware







Cone-Beam Artifacts



Iterative Image Reconstruction



$$x^{2} = y$$
Model
$$(x_{n} + \Delta x_{n})^{2} = y$$

$$x_{n}^{2} + 2x_{n}\Delta x_{n} + \mathbf{x}_{n}^{2} = y$$

$$x_{n}^{2} + 2x_{n}\Delta x_{n} \qquad \approx y$$

$$\Delta x_{n} = \frac{1}{2}(y - x_{n}^{2})/x_{n}$$

$$x_{n+1} = x_{n} + \Delta x_{n}$$
Update equation

This is an iterative solution.



Influence of Update Equation and Model	
$0.4 (3 - x_n^2) / x_n$	$0.5 (3 - x_n^{2.1})/x_n$
$x_0 = 1.$	$x_0 = 1.$
$x_1 = 1.8$	$x_1 = 2.$
$x_2 = 1.74667$	$x_2 = 1.67823$
$x_3 = 1.73502$	$x_3 = 1.68833$
$x_4 = 1.73265$	$x_4 = 1.68723$
$x_5 = 1.73217$	$x_5 = 1.68734$
$x_6 = 1.73207$	$x_6 = 1.68733$
$x_7 = 1.73206$	$x_7 = 1.68733$
$x_8 = 1.73205$	$x_8 = 1.68733$
	Jpdate Equation $0.4 (3 - x_n^2)/x_n$ $x_0 = 1.$ $x_1 = 1.8$ $x_2 = 1.74667$ $x_3 = 1.73502$ $x_4 = 1.73265$ $x_5 = 1.73217$ $x_6 = 1.73207$ $x_7 = 1.73206$ $x_8 = 1.73205$

 $x^2 = 3, \quad x_0 = 1, \quad x_{n+1} = x_n + \Delta x_n$



Kaczmarz's Method = ART



$$oldsymbol{f}_{
u+1} = oldsymbol{f}_{
u} + oldsymbol{R}^{\mathrm{T}} \cdot rac{oldsymbol{p} - oldsymbol{R} \cdot oldsymbol{f}_{
u}}{oldsymbol{R}^2 \cdot oldsymbol{1}}$$





Direct vs. Filtered Backprojection





Flavours of Iterative Reconstruction

• ART
$$oldsymbol{f}_{
u+1} = oldsymbol{f}_{
u} + oldsymbol{R}^{\mathrm{T}} \cdot rac{oldsymbol{p} - oldsymbol{R} \cdot oldsymbol{f}_{
u}}{oldsymbol{R}^2 \cdot 1}$$

• SART
$$\boldsymbol{f}_{\nu+1} = \boldsymbol{f}_{\nu} + \frac{1}{\boldsymbol{R}^{\mathrm{T}} \cdot \boldsymbol{1}} \boldsymbol{R}^{\mathrm{T}} \cdot \frac{\boldsymbol{p} - \boldsymbol{R} \cdot \boldsymbol{f}_{\nu}}{\boldsymbol{R} \cdot \boldsymbol{1}}$$

• MLEM
$$\boldsymbol{f}_{\nu+1} = \boldsymbol{f}_{\nu} \frac{\boldsymbol{R}^{\mathrm{T}} \cdot \left(e^{-\boldsymbol{R} \cdot \boldsymbol{f}_{\nu}}\right)}{\boldsymbol{R}^{\mathrm{T}} \cdot \left(e^{-\boldsymbol{p}}\right)}$$

• OSC
$$\boldsymbol{f}_{\nu+1} = \boldsymbol{f}_{\nu} + \boldsymbol{f}_{\nu} \frac{\boldsymbol{R}^{\mathrm{T}} \cdot \left(e^{-\boldsymbol{R} \cdot \boldsymbol{f}_{\nu}} - e^{-\boldsymbol{p}}\right)}{\boldsymbol{R}^{\mathrm{T}} \cdot \left(e^{-\boldsymbol{R} \cdot \boldsymbol{f}_{\nu}} \boldsymbol{R} \cdot \boldsymbol{f}_{\nu}\right)}$$

• and hundreds more ...



Bayesian Reconstruction = statistical reconstruction

- Finding an image f such that the probability of f given the projection data p, i.e. P(f|p), is maximized is difficult.
- Since we know from Bayes that

 $P(\boldsymbol{f}|\boldsymbol{p})P(\boldsymbol{p}) = P(\boldsymbol{p}|\boldsymbol{f})P(\boldsymbol{f})$

we may as well maximize P(p|f), because without further information the a priori probabilities introduce nothing but a positive factor of proportionality.

- If we have further information, e.g. on *f*, we may incorporate this prior knowledge and maximize the a posteriori probability P(p|f)P(f) instead.
- In log domain this becomes

$$f = \arg\min_{f} \left(L(p|f) + L(f) \right)$$



Objective Function: Gauß Model

Assume that the attenuation is Gaussian-distributed

$$\mathcal{L}(A) = \mathcal{N}(\sigma, \boldsymbol{r} \cdot \boldsymbol{f})$$

i.e. $P(A = a) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}(a - \mu)^2/\sigma^2}$ with $\mu = \boldsymbol{r} \cdot \boldsymbol{f}$.
Consequently, the likelihood for all N measured

• Consequently, the likelihood for all *N* measured signals is ($\mu_n = r_n \cdot f$):

$$P(\boldsymbol{A} = \boldsymbol{a}, \boldsymbol{f}) = \prod_{n} P(A_n = a_n)$$

• Before maximizing take the log, penalize roughness,

$$L(\boldsymbol{f}) = -\sum_{n} \left(\frac{a_n - \mu_n}{\sigma_n}\right)^2 - \beta R(\boldsymbol{f})$$

and then find the image f that maximizes L.



Gauß Model (continued)

This leads us to minimizing

$$(\boldsymbol{R}\cdot\boldsymbol{f}-\boldsymbol{a})^{\mathrm{T}}\cdot\boldsymbol{D}\cdot(\boldsymbol{R}\cdot\boldsymbol{f}-\boldsymbol{a})$$

which means solving

$$\boldsymbol{R}^{\mathrm{T}} \cdot \boldsymbol{D} \cdot (\boldsymbol{R} \cdot \boldsymbol{f} - \boldsymbol{a}) = \boldsymbol{0}$$

 This must be done numerically (e.g. Jacobi method) and the solutions are often of type

 $\boldsymbol{f}_{\nu+1} = \boldsymbol{f}_{\nu} + \operatorname{diag}(\boldsymbol{u}) \cdot \boldsymbol{R}^{\mathrm{T}} \cdot \operatorname{diag}(\boldsymbol{v}) \cdot (\boldsymbol{a} - \boldsymbol{R} \cdot \boldsymbol{f}_{\nu})$



Update Equation: Gauß Model

• ART
$$oldsymbol{f}_{
u+1} = oldsymbol{f}_{
u} + oldsymbol{R}^{\mathrm{T}} \cdot rac{oldsymbol{p} - oldsymbol{R} \cdot oldsymbol{f}_{
u}}{oldsymbol{R}^2 \cdot oldsymbol{1}}$$

• SART
$$\boldsymbol{f}_{\nu+1} = \boldsymbol{f}_{\nu} + \frac{1}{\boldsymbol{R}^{\mathrm{T}} \cdot \boldsymbol{1}} \boldsymbol{R}^{\mathrm{T}} \cdot \frac{\boldsymbol{p} - \boldsymbol{R} \cdot \boldsymbol{f}_{\nu}}{\boldsymbol{R} \cdot \boldsymbol{1}}$$

• and many more ...



Objective Function: Poisson Model

Assume that the intensities are Poisson-distributed

which means $P(I=i) = rac{\mu^i}{i!}e^{-\mu}$ with $\mu = I_0e^{-\boldsymbol{r}}\cdot\boldsymbol{f}$.

 $\mathcal{L}(I) = \mathcal{P}(I_0 e^{-\boldsymbol{r}} \cdot \boldsymbol{f})$

• Consequently, the likelihood for all *N* measured signals is $(\mu_n = I_0 e^{-r_n \cdot f})$:

$$P(\boldsymbol{I}=\boldsymbol{i},\boldsymbol{f}) = \prod_{n} P(I_{n}=i_{n}) = \prod_{n} \frac{\mu_{n}^{i_{n}}}{i_{n}!} e^{-\mu_{n}}$$

Before maximizing take the log, penalize roughness,

$$L(\boldsymbol{f}) = \sum_{n} (i_n \ln \mu_n - \mu_n) - \beta R(\boldsymbol{f})$$

and then find the image f that maximizes L.


• MLEM $f_{\nu+1} = f_{\nu} \frac{R^{T} \cdot (e^{-R \cdot f_{\nu}})}{R^{T} \cdot (e^{-p})}$

• OSC
$$\boldsymbol{f}_{\nu+1} = \boldsymbol{f}_{\nu} + \boldsymbol{f}_{\nu} \frac{\boldsymbol{R}^{\mathrm{T}} \cdot \left(e^{-\boldsymbol{R} \cdot \boldsymbol{f}_{\nu}} - e^{-\boldsymbol{p}}\right)}{\boldsymbol{R}^{\mathrm{T}} \cdot \left(e^{-\boldsymbol{R} \cdot \boldsymbol{f}_{\nu}} \boldsymbol{R} \cdot \boldsymbol{f}_{\nu}\right)}$$

• and many more ...



Iterative Reconstruction: Parameters

- Image/object representation
 - Pixel centers

$$f(x,y) = \sum f_m b(x - x_m, y - y_m)$$

- Blobs

Pixel area

- Sampling density (pixel size, pixel locations, ...)
- Forward model (forward projection)
 - Joseph-type, Bresenham-type, distance-driven-type, ...
 - Needle beam (infinitely thin ray), many needle beams per ray, ...
 - Beam shape (varying beam cross-section, angular blurring, ...)
 - Physical effects (beam hardening, scatter, motion, detector sensitivity, nonlinear partial volume effect, ...)

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Objective function, update equation

- Statistical model (Gaussian, Poisson, shifted Poisson, ...)
- Regularisation (edge-preserving, ...)
- Artifact reduction
- Inverse model (backprojection)
 - Transpose of forward model
 - Pixel-driven backprojection
 - Filtered backprojection

 $C(\boldsymbol{f}) = \left(\boldsymbol{R}\cdot\boldsymbol{f} - \boldsymbol{p}\right)^2$



Image Representation

•	•	•	•	•
•	•	•	•	•
•	•	•	•	•
•	•	•	•	•
•	•	•	•	•

 $b(x,y) = \bullet$



Image Representation



$$b(x,y) =$$



Image Representation



b(x,y) =



























Image Representation and Forward Model are Linked!



Joseph's forward projector



Iterative Reconstruction

- Aim: less artifacts, lower noise, lower dose
- Iterative reconstruction
 - Reconstruct an image.
 - Does the image correspond to the rawdata?
 - If not, reconstruct a correction image and continue.
- SPECT + PET are iterative for a long time!
- CT product implementations
 - ASIR (adaptive statistical iterative reconstruction, GE)
 - iDose (Philips)
 - IRIS (image reconstruction in image space, Siemens)
 - AIDR 3D (adaptive iterative dose reduction, Toshiba)
 - VEO, MBIR (model-based iterative reconstruction, GE)
 - IMR (iterative model reconstruction, Philips)
 - SAFIRE, ADMIRE (advanced modeled iterative reconstruction, Siemens)
 - FIRST (forward projected model-based iterative reconstruction solution, Toshiba)









- Rawdata regularization: adaptive filtering¹, precorrections, filtering of update sinograms...
- Inverse model: backprojection (R^{T}) or filtered backprojection (R^{-1}). In clinical CT, where the data are of high fidelity and nearly complete, one would prefer filtered backprojection to increase convergence speed.
- Image regularization: edge-preserving filtering. It may model physical noise effects (amplitude, direction, correlations, ...). It may reduce noise while preserving edges. It may include empirical corrections.
- Forward model (R_{phys}) : Models physical effects. It can reduce beam hardening artifacts, scatter artifacts, cone-beam artifacts, noise, ...

¹M. Kachelrieß et al., Generalized Multi-Dimensional Adaptive Filtering, MedPhys 28(4), 2001





Conventional FBP with rawdata denoising (all vendors)

ASIR, ASIR-V (Ge), AIDR3D (Toshiba), IRIS (Siemens), iDose (Philips), SnapShot Freeze (GE), iTRIM (Siemens)



M. Kachelrieß. Current Cardiovascular Imaging Reports 6:268–281, 2013







Courtesy of Dr. Jiang Hsieh, GE Healthcare Technologies, WI, USA.



Filtered Backprojection





Courtesy of Dr. Waldemar Hosch, Zürich, Switzerland.



FBP





Courtesy of Dr. Thomas Köhler, Philips, Germany.





Courtesy of Siemens Healthcare, Forchheim, Germany.



Filtered Backprojection







Courtesy of Dr M Chen, NHLBI, National Institutes of Health, USA





Courtesy of Dr. Patrik Rogalla, UHN, Toronto, Canada



SIEMENS

Advantages of SAFIRE versus Linear Noise Reduction





Conventional reconstruction

at 100% dose



Iterative reconstruction and restoration

at 40% dose





Conventional reconstruction at 100% dose

Iterative reconstruction and restoration at 40% dose





Conventional reconstruction

at 100% dose



Iterative reconstruction and restoration

at 40% dose





Vendor's Improvements in Iterative Reconstruction



Vendor's Improvements in Iterative Reconstruction



Extremely low dose case: $CTDI_{vol} = 0.04 \text{ mGy}$, $DLP = 1.64 \text{ mGy} \cdot \text{cm}$, $D_{eff} = 0.025 \text{ mSv}$



Vendor's Improvements in Iterative Reconstruction



Toshiba Aquilion ONE VISION FIRST Edition

Akagi et al. Full Iterative Reconstruction Optimized for Specific Organs -Principle and Capabilities. RSNA 2015.



Usual Assumption: CT is Linear and Translation Invariant

- PSF and MTF are well-defined
- Noise is well-defined
- Noise and spatial resolution are related
- Parameters are valid for all objects
- Simple phantoms can be used to assess image quality



Simple Example 1

(Taken at the Siemens Somatom Flash DSCT Scanner)

Semiantropomorphic phantom

- 20 cm × 30 cm thorax phantom of 20 cm length with 2.5 cm water extension ring, totalling to 25 cm × 35 cm size
- 10 cm QRM 3D medium contrast insert with 40 HU background and 20 HU lesions (at 120 kV)

Scan and recon parameters

- 128 × 0.6 mm collimation
- 120 kV
- p = 0.6
- $t_{\rm rot} = 1.0 \, {\rm s}$
- $S_{\rm eff} = 0.6 \, \rm mm$
- 1 full dose scan with 1100 mAs_{eff}
- 25 low dose scans with 44 mAs_{eff} each
- FBP (= analytical): B30s, B50s
- SAFIRE (= iterative): I30s and I50s, strengths 3 and 5
- Averaging of 25 low dose scans after reconstruction
- Mean±StdDev in large medium contrast lesion
- Display at C = 50 HU and W = 100 HU



















Simple Example 2

- Same phantom as in example 1
- Same scans as in example 1
- Calculation of sigma images from the 25 independent samples
 - Compute unbiased estimator for the sample variance for each pixel
 - Take the square-root of each pixel's estimated variance





MK1 Anmerkung von Stefan S.: Warum sind bei dem sigma FBP Bild die Kanten außen sichtbar und zur Lunge hin verwischt?

Idee: außen wirkt das Gibbs Phänomen und die CT-Werte werden nach unten hin abgeschnitten (-1024 HU. Die Lunge hat einen CT-Wert von ~ -800 HU, dort wird nichts abgeschnitten) Prof. Dr. Marc Kachelrieß; 24.06.2015
Simple Example 3

(Taken at the Siemens Somatom Flash DSCT Scanner)

- Abdomen phantom + small fat ring
- Tube voltage U = 120 kV
- Slice thickness S_{eff} = 0.6 mm
- Pitch *p* = 0.6
- Variation of the effective tube current
 - mAs_{eff} = 100 mAs ... 550 mAs
 - DLP = 57 ... 312 mGy⋅cm
- Noise was measured in VOIs



Image Noise vs. mAs_{eff}



Conclusions on the Simple Examples and General Comments

- The (SAFIRE) iterative reconstruction
 - reduces noise in low and medium contrast regions
 - reduces spatial resolution in low and medium contrast regions
 - preserves noise in high contrast regions (edges)
 - preserves spatial resolution in high contrast regions (edges)
 - shows the conventional square-root relation of image noise and dose
- Other iterative standard reconstruction algorithms
 - also attempt to reduce noise and preserve resolution
 - will also not reduce noise at edges
 - may behave different in detail
 - may deviate more or less from the square-root behaviour
- Future iterative reconstructions algorithms
 - may compensate for motion
 - may use stronger a priori knowledge (e.g. dictionaries)



Analysis of GE's MBIR (Veo) Iterative Reconstruction Algorithm

Statistical model based iterative reconstruction (MBIR) in clinical CT systems. Part II. Experimental assessment of spatial resolution performance

Ke Li

Department of Medical Physics, University of Wisconsin-Madison, 1111 Highland Avenue, Madison, Wisconsin 53705 and Department of Radiology, University of Wisconsin-Madison, 600 Highland Avenue, Madison, Wisconsin 53792

John Garrett and Yongshuai Ge Department of Medical Physics, University of Wisconsin-Madison, 1111 Highland Avenue, Madison, Wisconsin 53705

Guang-Hong Chen^{a)} Department of Medical Physics, University of Wisconsin-Madison, 1111 Highland Avenue, Madison, Wisconsin 53705 and Department of Radiology, University of Wisconsin-Madison, 600 Highland Avenue, Madison, Wisconsin 53792

(Received 8 March 2014; revised 9 May 2014; accepted for publication 2 June 2014; published 23 June 2014)



Contrast Dependency of the PSF (of GE's FBP and Veo Algorithms)





Dose Dependency of the PSF (of GE's FBP and Veo Algorithms)







Conclusions on Li et al. (Veo Algorithm)

- Our previous findings (from the simple examples) are confirmed.
- Spatial resolution is a function of
 - location
 - contrast
 - dose
 - ...



Thank You!



Conference Chair Marc Kachelrieß, German Cancer Research Center (DKFZ), Heidelberg, Germany

This presentation will soon be available at www.dkfz.de/ct. Parts of the reconstruction software were provided by RayConStruct[®] GmbH, Nürnberg, Germany.